CHAPTER 8
BOOLEAN SIMPLIFICATION

This chapter will be devoted toward understanding how output expressions may be simplified by various methods. By proper application of simplification techniques the systems designer can be sure a circuit is in its simplest algebraic form. It is obvious that the more simple a circuit is the fewer components are needed and the cheaper the cost will be to construct the circuit.

It should be understood that there are cases where the simplest electronic circuit is not the result of the simplest algebraic expression; however, by application of Boolean algebra to manipulate a simplified expression, a designer can obtain a circuit that is simplest from an electronic viewpoint.

After a designer determines his Boolean function by means of the minterm or maxterm expression, he must know whether this expression may be simplified. By simplified, we mean that another expression may be determined that will represent the same function with less equipment. For example, the designer may arrive at the function $f(A,B,C) = A\bar{B} + B\bar{C} + \bar{B}C + \bar{A}B$. This function can be simplified to give $f(A,B,C) = A\bar{B} + B\bar{C} + \bar{A}C$ which may be easier to construct.

ORDER OF EXPRESSION

When describing Boolean functions, it is often necessary to identify them as to their order. The order is defined, for example, so that the cost of the logic circuit may be determined without constructing the circuit. Higher order expressions generally result in more cost to construct the circuits.

To determine the order of a Boolean expression, we must first inspect the quantity within the parentheses. If this quantity contains only an AND operation(s), or only an OR operation(s), this quantity is first order. If the quantity contains both an AND and an OR operation(s), it is considered a second-order quantity.

The next step is to consider the relationship of the quantity within the parentheses and the other variables within the brackets of the expression. Again, if the parenthesized quantity is combined with the other bracketed variables with either an AND operation(s) or an OR operation(s), the order is increased accordingly. This process is continued until the final order of the expression is obtained.

To find the order of the expression

$$[(AB + C)\,D + E]\,F + G$$

first consider the parenthesized quantity

$$(AB + C)$$

This quantity contains an AND operation and an OR operation; therefore, it is second-order. Now consider the quantity in brackets; that is,

$$[(AB + C)\,D + E]$$

The parenthesized quantity (second-order) is combined with an AND and an OR operation; therefore, the quantity in brackets is fourth-order. Finally, the quantity in brackets (fourth-order) is combined with an AND and an OR operation in

$$[(AB + C)\,D + E]\,F + G$$

and the entire expression is then sixth-order. To find the order of the expression

$$[(AB)(CD + EF) + G]$$

begin with the parenthesized quantity which has the highest order; that is

$$(CD + EF)$$

It contains both AND and OR operations and is therefore second-order. The quantity in brackets contains a second-order quantity combined with both AND and OR operations and is a fourth-order expression.

SIMPLIFICATION

Since one input signal may accomplish the function of another, we may, in many cases, eliminate the superfluous signal. We use the basic laws of Boolean algebra in order to eliminate parts of expressions without changing the logic state of the output.

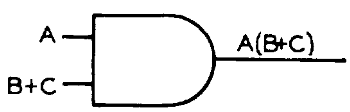
It is important to recognize whether one part of an expression is equal to another; that is, by recalling the commutative law we find that

$$ABC = BAC$$

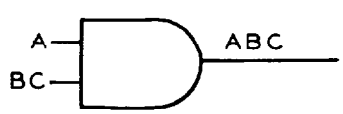
and

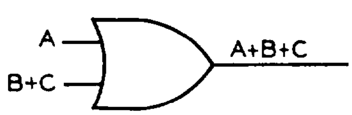
$$A(B + C) = (C + B)\,A$$

Previously, when writing output expressions in order to reproduce circuit diagrams, we use parentheses in all cases except for ANDed inputs to OR symbols. In simplification, we use parentheses only when we have an ORed input to an AND symbol. That is,

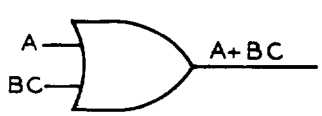


The other three cases are shown without parentheses as follows:





and



To simplify

$$(AB + C) + D + EF$$

we write

$$AB + C + D + EF$$

The idempotent law states that

$$AA = A$$

and

$$A + A = A$$

This law is used to simplify expressions as follows:

$$(AB)(AB) = AB$$

and

$$\begin{aligned} A+B+A+C+B+D &= A+A+B+B+C+D \\ &= A+B+C+D \end{aligned}$$

and

$$A\bar{B}\bar{C} + A\bar{B}\bar{C} = A\bar{B}\bar{C}$$

PROBLEMS: Simplify the following:

1. (AA)(BB)
2. AB + (AB + CD)
3. $(AB + C)(D\bar{E})(\bar{E}D)(C + BA)$
4. (ABC + DE) + CBA

ANSWERS:

1. AB
2. AB + CD
3. $(AB + C)(D\bar{E})$
4. ABC + DE

When simplifying expressions which contain negations, the use of the law of double negation is used. This law indicates that whenever two negation bars of equal length cover the same letter or expression, both bars may be removed without affecting the value of the expression. To simplify the expression

$$\overline{\overline{AB}}$$

we write

$$\overline{\overline{AB}} = AB$$

and the expression

$$\overline{\overline{AB}} + \overline{\overline{C}} = \overline{AB} + C$$

Notice that we removed only two bars from above AB.

To simplify the expression

$$(\overline{\overline{A + B}} + C) + \overline{\overline{C}} + (A + B)$$

we use the laws which we have discussed to this point and write

$$\overline{\overline{A + B}} = A + B$$

and

$$\overline{\overline{C}} = C$$

therefore

$$(\overline{\overline{A + B + C}}) + \overline{\overline{C}} + (\overline{\overline{A + B}})$$

equals

$$(A + B + C) + C + (A + B)$$

then by removing the parentheses and applying the commutative law write

$$A + A + B + B + C + C$$

then by the idempotent law this equals

$$A + B + C$$

PROBLEMS: Simplify the following expressions.

1. $(A\overline{\overline{BC}} + D)\ E + \overline{\overline{\overline{F}}}$
2. $(\overline{\overline{AB}}C + D)\ (D + \overline{\overline{B}}\ AC)$

ANSWERS:

1. $(ABC + D)\ E + \overline{F}$
2. $ABC + D$

In the discussion of the complementary law, the logic state of the output is considered. This law indicates that when any letter or expression is ANDed with its complement, the output is 0. When any letter or expression is ORed with its complement, the output is 1; that is,

$$A\overline{A} = 0$$

and

$$A + \overline{A} = 1$$

The logic state of

$$(A + B)\ \overline{(A + B)} = 0$$

and the logic state of

$$AB + \overline{AB} = 1$$

To simplify the expression

$$\overline{AB\overline{C}} + A(\overline{C}B)$$

we write

$$\overline{AB\overline{C}} + A(\overline{C}B)$$

$$\overline{AB}C \ + \ AB\overline{C}$$

and notice that

$$\overline{AB}C$$

is the complement of

$$AB\overline{C}$$

therefore

$$\overline{AB\overline{C}} + A(\overline{C}B) = 1$$

PROBLEMS: Simplify the following expression.

1. $A\overline{A}\overline{\overline{A}}$
2. $(A + \overline{\overline{\overline{A}}})\ A\overline{A}$
3. $\overline{A(BC)\ D} + (AB)\overline{\overline{(CD)}}$

ANSWERS:

1. 0
2. 0
3. 1

Since intersection indicates the AND operation, we concern ourselves with the law of intersection for simplification of ANDed operations. The law of intersection is

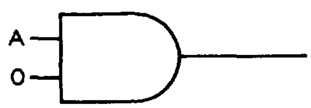
$$A \cdot 1 = A$$

and

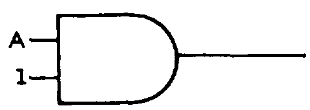
$$A \cdot 0 = 0$$

Therefore, whenever we have an input of 1 ANDed with an input A, the output will have the same binary value as A. Also, if the input 0 is ANDed with A, the output will have the binary value of 0.

This is best illustrated by the diagrams. If two inputs to an AND circuit are



the output will be 0. If the two inputs are



then the output will take the value of A; that is, if A is 1, the output is 1; and if A is 0, the output is 0.

To simplify the expression

$$(AB + C)\,0$$

we need only consider this as straightforward multiplication and write the answers as 0.

To simplify the expression

$$(AB + C)(D + E) \cdot 1$$

we apply the same logic and write $(AB + C)(D + E)$.

When we desire to simplify an expression such as

$$(A + B)\overline{(A + B)}(C + D)$$

we use the complementary law to find

$$(A + B)\overline{(A + B)} = 0$$

and the law of intersection to find

$$0(C + D) = 0$$

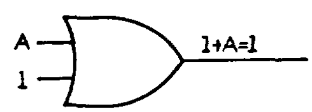
PROBLEMS: Simplify the following expressions.

1. $\bar{A}(B + \bar{B})$
2. $\bar{\bar{A}} + (B + \bar{B})A$
3. $(A + \bar{A})(BC + DE)$

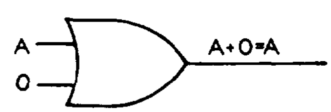
ANSWERS:

1. $\bar{A}$
2. A
3. $(BC + DE)$

The law of union is used in simplifying expressions in somewhat the same manner as the law of intersection is used. The difference between the two laws is that the law of union is considered as straightforward addition; that is,



and



To simplify the expression

$$A + B + 1$$

we write the answer as 1 because we may have only 0 or 1 as the output; and since we are using the process of addition, the values that A and B take have no effect on the output.

DeMorgan's theorem is a useful tool in simplifying Boolean algebra expressions. It is presented basically in the following two equations:

$$\overline{A + B} = \bar{A}\bar{B}$$

$$\overline{AB} = \bar{A} + \bar{B}$$

These equations indicate that in the process of simplification whenever you split or join vincula you change the sign; that is, AND to OR, or OR to AND.

When applying DeMorgan's theorem to the expression

$$\overline{A + B + C}$$

write

$$\bar{A}\,\bar{B}\,\bar{C}$$

Further examples are:

$$\bar{A} + \bar{B} + \bar{C} = \overline{ABC}$$

and

$$\bar{A}\,\bar{B}\,\bar{C} = \overline{A + B + C}$$

It should be noted that when you change signs in an expression you must group the same letters that were originally grouped; that is,

$$\overline{AB + C} = (\bar{A} + \bar{B})\bar{C}$$

Other examples are:

$$(\bar{A} + \bar{B})\bar{C} = \overline{AB + C}$$

and

$$(\bar{A} + \bar{B})\,\bar{C} + \bar{D} = \overline{(AB + C)\,D}$$

In cases where the vinculum covers part of an expression, the signs under the vinculum change while the signs outside the vinculum do not change; that is,

$$A + \overline{B(C + D) + E} = A + (\bar{B} + \bar{C}\bar{D})\bar{E}$$

and

$$\overline{(AB + C)D} + \overline{EF} = (\bar{A} + \bar{B})\bar{C} + \bar{D} + \bar{E} + \bar{F}$$

Using DeMorgan's theorem to simplify

$$AB + CD + \bar{A} + \bar{B}$$

we write

$$\bar{A} + \bar{B} = \overline{AB}$$

then substituting we have

$$AB + CD + \overline{AB}$$

and

$$AB + \overline{AB} = 1$$

therefore

$$AB + CD + \overline{AB}$$

$$= 1 + CD$$

$$= 1$$

We may also split a vinculum to simplify an expression as follows:

$$A + B + CD + \overline{AB}$$

$$= A + B + CD + \bar{A} + \bar{B}$$

$$= A + \bar{A} + B + \bar{B} + CD$$

$$= 1 + 1 + CD$$

$$= 1$$

In some cases it may be necessary to manipulate one part of an expression so that it is the complement of another part of the expression. This may be accomplished in the following manner.

If, when simplifying the expression

$$[A + \overline{B(C + D)}][\bar{A}B(C + D)] + E$$

we choose to split the vinculum in the first parentheses, we have

$$A + \bar{B} + \bar{C}\bar{D}$$

which is not in a more simple form. Therefore, we elect to add two vincula over A and have

$$\bar{\bar{A}} + \overline{B(C + D)}$$

then we join the vincula to find

$$\overline{\bar{A}\ B(C + D)}$$

which is the complement of the expression in the second parentheses. Then,

$$[\overline{\bar{A}\ B(C + D)}][\bar{A}\ B(C + D)] = 0$$

and

$$0 + E = E$$

When simplifying expressions involving vincula, it must be understood that if any letter in an expression has more than one vinculum over it, the expression is not in the simplest form; that is, the expression

$$\overline{\bar{A} + BC}$$

is not in the simplest form. In this case we would split the long vinculum and have

$$\bar{\bar{A}}\ (\bar{B} + \bar{C})$$

which simplifies to

$$A\ (\bar{B} + \bar{C})$$

In order to simplify the expression

$$\overline{\overline{\overline{(A + B)}.C} + \overline{(D + \bar{E})\ F}}$$

by the methods discussed would require many steps. An easier method is to remember that if one vinculum is removed the operation changes, and if two vincula are removed the operation remains the same; that is,

$$\bar{\cdot} = +$$

$$\bar{\bar{\cdot}} = \cdot$$

$$\bar{+} = \cdot$$

$$\bar{\bar{+}} = +$$

$$\bar{\bar{\bar{+}}} = \cdot$$

Using this technique to simplify the expression

$$\overline{A\ \overline{(B + C)}}$$

we think

$$\bar{A}\bar{\cdot}(\bar{\bar{B}}\ \bar{\bar{+}}\ \bar{\bar{C}})$$

and write

$$\bar{A} + B + C$$

To simplify the expression

$$\overline{A\ \bar{B}(C + \overline{DE}) + F}$$

we think

$$\bar{A}\bar{\cdot}\bar{\bar{B}}\bar{\cdot}[\bar{C}\ \bar{+}\ \bar{\bar{D}}\bar{\bar{\cdot}}\bar{\bar{E}}]\ \bar{+}\ \bar{F}$$

and write

$$[\bar{A} + B + [\bar{C} \cdot (D \cdot E)]] \cdot \bar{F}$$

and then

$$(\bar{A} + B + \bar{C}\ D\ E)\ \bar{F}$$

We used the double brackets and parentheses in order to maintain the proper groupings.

PROBLEMS: Simplify the following expression.

1. $\overline{(A + \bar{A})(\overline{BC} + \bar{C} + \bar{B})}$

2. $\overline{\overline{(AB + C)}(\bar{A} + \bar{B})\ \bar{C}}$

ANSWERS:

1. BC

2. AB + C

To this point in our discussion we have determined there are two ways in which DeMorgan's theorem may be applied in simplifying expressions. First, we may join vincula to form the complement of a term or expression. Second, we may split vincula and then apply other laws to further simplify the expression.

In many cases an expression may not be simplified in one form while simplification of the expression may be accomplished if the expression is in another form. The distributive law is one which enables us to change the form of an expression. The distributive law contains two identities. These are:

$$A\ (B + C) = AB + AC$$

and

$$A + BC = (A + B)(A + C)$$

Notice that the second identity is true for Boolean algebra but is not true for ordinary algebra. The following conversions of expressions from one form to another form is given for understanding the distributive law.

$$A\,(B + C + D) = AB + AC + AD$$

$$AB\,(C + D) = ABC + ABD$$

and

$$A + BC = (A + B)(A + C)$$

$$A + BCD = (A + B)(A + C)(A + D)$$

Notice in the conversion that either side of the equations may be changed to the other form. That is,

$$ABC + ABD = AB\,(C + D)$$

and

$$(A + B + C)(A + E + F) = A + (B + C)(E + F)$$

In ordinary algebra the first of these equations was called carryout the distributive law and the last two equations were called factoring.

The expression

$$AB + A\overline{B}$$

cannot be simplified in its present form but by application of the distributive law we have

$$\begin{aligned} AB + A\overline{B} &= A\,(B + \overline{B}) \\ &= A \cdot 1 \\ &= A \end{aligned}$$

Also in this category is

$$\begin{aligned} (A + \overline{B})(A + B) &= A + (\overline{B}\,B) \\ &= A + 0 \\ &= A \end{aligned}$$

PROBLEMS: Convert the form of each of the following expressions to another form.

1. $AB + CDE$
2. $ABC + ABD + ACD$
3. $(A + B)(A + D)$
4. $(A + BC)(BC + D + E)$

ANSWERS:

1. $(AB + C)(AB + D)(AB + E)$
2. $A\,(BC + BD + CD)$
3. $A + BD$
4. $BC + A\,(D + E)$

In the previous discussion of the distributive law, the change of the form of the expressions is used to aid simplification. The final determination of which form to use is made by finding which form requires fewer gates. There are no hard-and-fast rules which may be used to determine which form is more simple.

For our discussion we will consider the more simple form of an expression as having no vinculum extending over more than one letter and having no parentheses.

PROBLEMS: Simplify the following expressions.

1. $AB + \overline{AB}C$
2. $ABC + AB\overline{C}$
3. $AB\,(AE + \overline{B} + C\overline{A})$

ANSWERS:

1. $AB + C$
2. AB
3. ABE

The law of absorption is very closely related to the distributive law in that we use the distributive law to show the law of absorption. The law of absorption is shown by

$$A + AB = A$$

and

$$A\,(A + B) = A$$

If we take

$$A + AB$$

and factor out an A we have

$$A\,(1 + B)$$

$$= A \cdot 1$$

$$= A$$

and also

$$A\,(A + B)$$

$$= AA + AB$$

$$= A + AB$$

where we used the distributive law.

Actually, the law of absorption eliminates terms of an expression which are not needed. This may be seen in the following.

In the expression

$$ABC + AB$$

if A and B each have the value of 1, then the output value is 1 regardless of the value of the term ABC. Therefore, the term ABC is not necessary because the only values which will make ABC equal 1 is for A, B, and C to each have the value of one and we have already agreed the output is 1 if A and B are equal to 1.

When we use the law of absorption to simplify this expression, we have

$$ABC + AB$$

$$= AB\,(C + 1)$$

$$= AB$$

PROBLEMS: Simplify the following expressions.

1. $AB + ABC + ABCD$
2. $AB + ABC + A$
3. $AB + CD + ABE$

ANSWERS:

1. AB
2. A
3. AB + CD

The law of absorption is also used to simplify expressions of the type

$$\bar{\bar{A}}\,\overline{\bar{A}\,B}$$

To simplify this expression we write

$$\bar{\bar{A}}\,\overline{\bar{A}\,B} = \bar{\bar{A}}\,(\bar{\bar{A}} + \bar{B})$$

$$= A\,(A + \bar{B})$$

$$= A + A\,\bar{B}$$

$$= A\,(1 + \bar{B})$$

$$= A$$

Also, to simplify

$$A\,(B + C + \overline{\bar{A} + \bar{D}})\,D$$

we write

$$A\,(B + C + \overline{\bar{A} + \bar{D}})\,D$$

$$= A\,(B + C + AD)\,D$$

$$= A\,D(B + C + AD)$$

$$= ABD + ACD + AD$$

$$= AD\,(B + C + 1)$$

$$= AD \cdot 1$$

$$= AD$$

PROBLEMS: Using any or all of the basic laws discussed, simplify the following expressions.

1. $AB + CD\bar{D} + \overline{\overline{BA}}$
2. $A + B + AC\bar{A}C$
3. $(\bar{B}B + AA)\,C$
4. $ABC\,(\bar{B} + B)$

ANSWERS:

1. AB

2. A + B

3. AC

4. ABC

VEITCH DIAGRAMS

There are many cases in which simplification of an expression is so involved that it becomes impractical to attempt the simplification by algebraic means. This situation may be averted by the use of the Veitch diagram. Veitch diagrams provide a very quick and easy way for finding the simplest logical equation needed to express a given function or expression. A Veitch diagram is a block of squares on which you plot an expression.

As previously mentioned, we will consider an expression as being in simplified form when no vinculum extends over more than one letter and the expression contains no parentheses. This process results in an expression in minterm form. Recall that a minterm is the symbolic product of a given number of variables; that is,

$$ABC$$

is a minterm of three variables and

$$ABCD$$

is a minterm of four variables.

An expression is in minterm form if it is composed only of minterms connected by the operation OR sign. An example of a minterm form expression is

$$AB + C + \bar{D}EF$$

while

$$AB + C + D(A + C)$$

is not in minterm form.

In order to place any expression in minterm form we need only split or remove vincula, remove parentheses, and simplify within the term. When the expression is in minterm form, further simplification is unnecessary if Veitch diagrams are to be used; that is, to convert the expression

$$\overline{A + B} + RS$$

to minterm forms, we write

$$\overline{A + B} + RS$$

$$= \bar{A}\bar{B} + RS$$

which is in minterm form.

PROBLEMS: Convert the following expressions to minterm form.

1. $A + B + \overline{CD}$

2. $\bar{A}\bar{B}C + D(E + F)$

3. $\overline{\overline{AB}} + CD + \bar{\bar{E}}B + \overline{\overline{BCD}} + \bar{\bar{\bar{E}}}$

4. $(AB + C)B + D(E + \bar{D})$

ANSWERS:

1. $A + B + \bar{C} + \bar{D}$

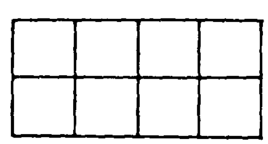
2. $\bar{A}\bar{B}C + DE + DF$

3. $AB + CD + EB + BC + \bar{D} + \bar{E}$

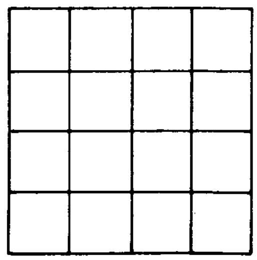
4. $AB + BC + DE$

In order to form a Veitch diagram it is necessary to know the number of possible minterms which may be formed from the variables in the expression to be simplified. To determine the number of minterms, raise 2 to the power of the number of variables; that is, if we have three variables, then we have 2^3 minterms possible.

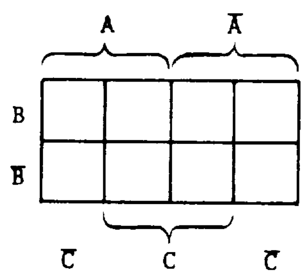
When we construct a Veitch diagram, there is one square for each minterm. A three-variable Veitch diagram is drawn as



and a four-variable Veitch diagram is drawn as

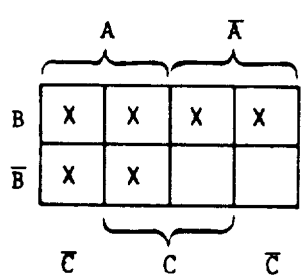


We label a Veitch diagram of eight squares as



Notice that half of the squares are assigned to each variable and the other half of the squares are assigned to the complements of the variables. Also, each variable overlaps every other variable and every complement except its own.

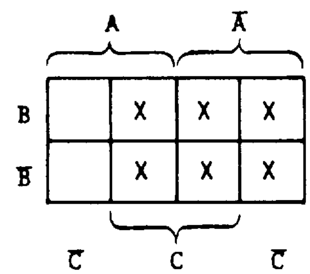
When we plot an expression such as A + B, we place an X in every square that is A and in every square that is B; that is,



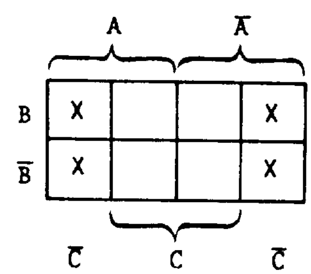
If we desire to plot the expression

$$\bar{A} + C$$

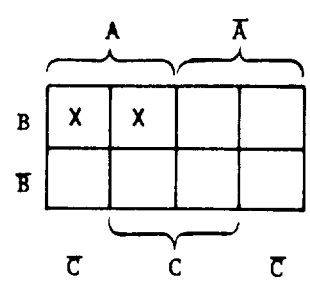
we write



and to plot $\bar{C}$ we write



When we plot a term such as AB, we plot only the squares common to both A and B; that is,



and when we plot a term such as $AB\bar{C}$, we write an X in the squares common to all those variables; that is,

	A	A	$\bar{A}$	$\bar{A}$
B	X			
$\bar{B}$				
	$\bar{C}$	C	C	$\bar{C}$

To plot an entire expression we plot each term on the same diagram. To plot the expression

$$AB + CB + \bar{A}BC$$

we write

	A	A	$\bar{A}$	$\bar{A}$
B	X	X	X	
$\bar{B}$				
	$\bar{C}$	C	C	$\bar{C}$

PROBLEMS: Plot the following expressions on Veitch diagrams.

1. $BC + A$
2. $\bar{A}C + \bar{B}\bar{C}$
3. $\bar{C}B + A\bar{C} + \bar{B}C + B\bar{A}$

ANSWERS:

1.

	A	A	$\bar{A}$	$\bar{A}$
B	X	X	X	
$\bar{B}$	X	X		
	$\bar{C}$	C	C	$\bar{C}$

2.

	A	A	$\bar{A}$	$\bar{A}$
B			X	
$\bar{B}$	X		X	X
	$\bar{C}$	C	C	$\bar{C}$

3.

	A	A	$\bar{A}$	$\bar{A}$
B	X			X
$\bar{B}$	X	X	X	X
	$\bar{C}$	C	C	$\bar{C}$

It should be noted that when plotting squares common to variables of a minterm a single variable term occupies four squares, a two-variable term occupies two squares, and a three-variable term occupies one square.

To extract the simplest expression from a Veitch diagram we look for, in order, four plotted squares which may be described by a one-variable term, two plotted squares which may be described by a two-variable term, and then one plotted square described by a three-variable term.

EXAMPLE: Simplify the following expression by use of a Veitch diagram.

$$\bar{A}BC + \bar{A}B\bar{C} + \bar{A}\bar{B}$$

SOLUTION: First we plot the terms of the expression or a Veitch diagram by writing

	A	A	$\bar{A}$	$\bar{A}$
B			X	X
$\bar{B}$			X	X
	$\bar{C}$	C	C	$\bar{C}$

then we follow the previous instructions and find the single variable term which expresses the plotted squares in as few terms as possible to be $\bar{A}$. Therefore, the expression

$$\bar{A}BC + \bar{A}B\bar{C} + \bar{A}\bar{B}$$

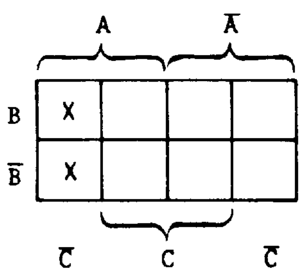
when simplified is

$$\bar{A}$$

EXAMPLE: Simplify by Veitch diagram the expression

$$AB\bar{C} + A\bar{B}\bar{C}$$

SOLUTION: Plot the diagram by writing

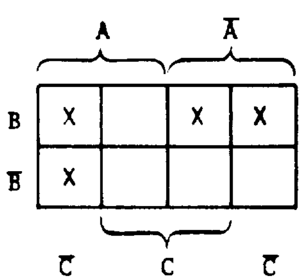


then express the two plotted squares by the fewest terms possible which is $A\bar{C}$.

EXAMPLE: Use the Veitch diagram to simplify the expression

$$AB\bar{C} + A\bar{B}\bar{C} + \bar{A}BC + \bar{A}B\bar{C}$$

SOLUTION: Write

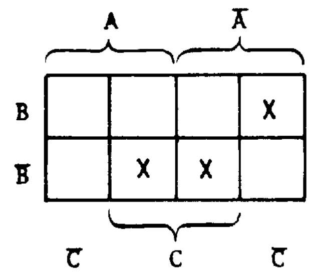


and the description of the plots in the fewest terms possible is

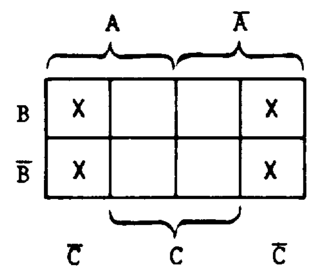
$$A\bar{C} + \bar{A}B$$

PROBLEMS: Describe the following Veitch diagrams in as few terms as possible.

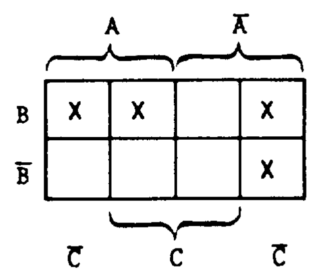
1.



2.



3.

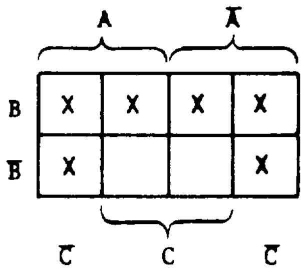


ANSWERS:

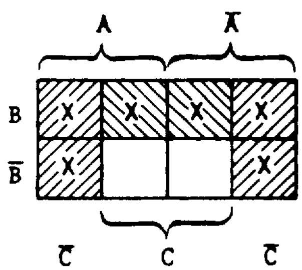
1. $\bar{B}C + \bar{A}B\bar{C}$
2. $\bar{C}$
3. $AB + \bar{A}\bar{C}$

When describing the plotted Veitch diagram, you should determine whether any plotted squares could be described twice. If so, this will, in many cases, result in the simplest description.

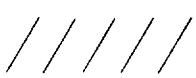
EXAMPLE: Describe the following Veitch diagram in as few variables in each term as possible.



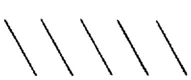
SOLUTION: We could consider the plots as shown



and write

 $= \bar{C}$

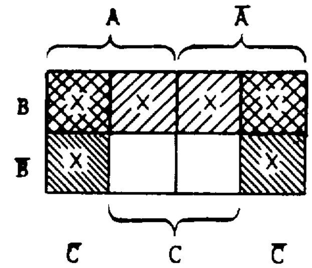
and

 $= BC$

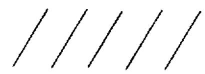
then the description would be

$$\bar{C} + BC$$

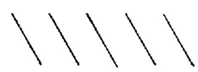
If we could consider the plots as



where

 $= B$

and

 $= \bar{C}$

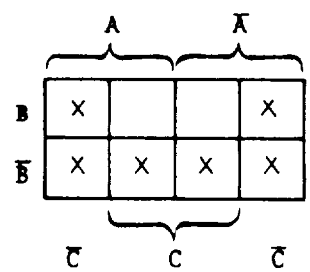
then the description would be

$$B + \bar{C}$$

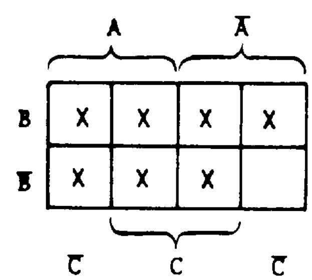
which is more simple than $\bar{C} + BC$ because of fewer variables in one term.

PROBLEMS: Describe the following diagrams by the simplest expression possible.

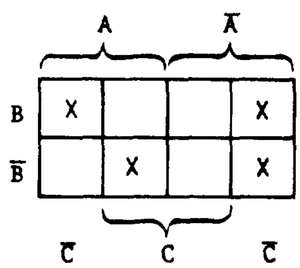
1.



2.



3.



ANSWERS:

1. $\bar{B} + \bar{C}$

2. $A + B + C$

3. $\bar{A}\bar{C} + B\bar{C} + A\bar{B}C$

At this point it should be obvious that in order to simplify a Boolean expression by use of the Veitch diagram process it is necessary to proceed as follows:

1. Write the expression in minterm form.

2. Plot a Veitch diagram for the variables involved.

3. Extract the simplest expression from the diagram.

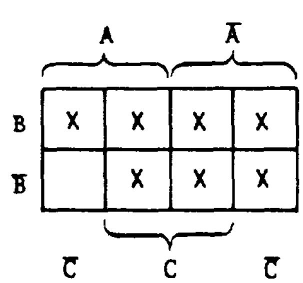
EXAMPLE: Simplify the following expression by use of a Veitch diagram.

$$AB + C + \bar{C}(\bar{A}B + \bar{A}\,\bar{B})$$

SOLUTION: Employ step (1) and write

$$AB + C + \bar{C}(\bar{A}B + \bar{A}\,\bar{B}) = AB + C + \bar{A}B\bar{C} + \bar{A}\bar{B}\bar{C}$$

Follow step (2) and write



then extract the simplest expression from the plotted Veitch diagram to find

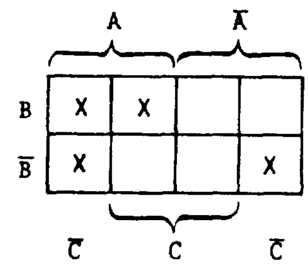
$$\bar{A} + B + C$$

EXAMPLE: Simplify the expression

$$AB + \bar{B}\,\bar{C} + A\bar{C}$$

by use of the Veitch diagram

SOLUTION: Write

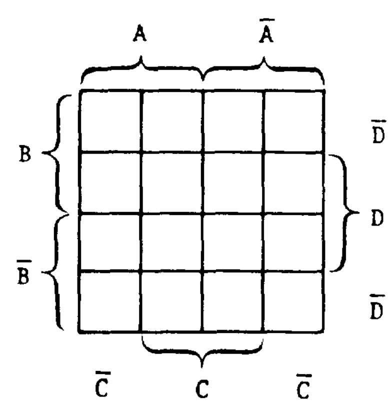


then extract the expression

$$\bar{B}\,\bar{C} + AB$$

To understand the power of the Veitch diagram method of simplification, the reader should attempt the simplification of $AB + \bar{B}\,\bar{C} + A\bar{C}$ by the use of the laws of Boolean algebra.

In the event that an expression contains four variables, we must determine the number of squares of the Veitch diagram by using 2^n where n is the number of variables. This results in 16 squares. We label the Veitch diagram as

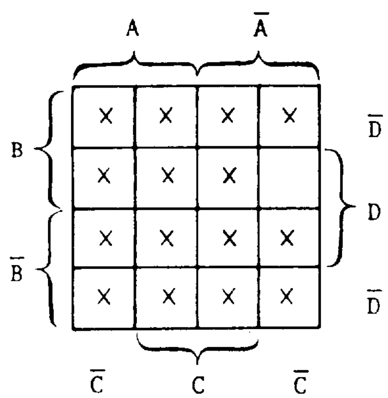


The same process is followed with 16 squares as was followed with 8 squares. The difference is that with 16 squares a single variable has 8 squares assigned, a two-variable term has 4 squares assigned, a three-variable term has 2 squares assigned, and a four-variable term is described by a single square.

To plot the expression

$$A + \bar{B} + C + \bar{D}$$

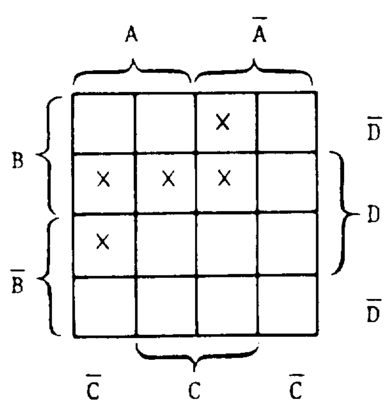
we write



and to plot the expression

$$\bar{A}BC + BCD + A\bar{C}D$$

we write



PROBLEMS: Plot the following expressions on a Veitch diagram.

1. $B + D$

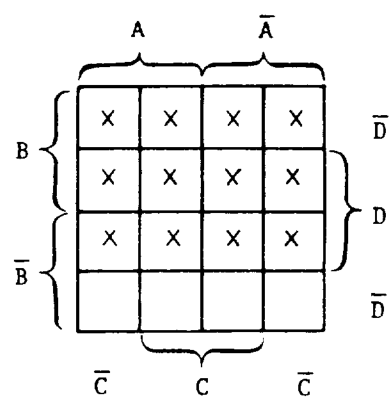
2. $\bar{A}B + \bar{C}\bar{D}$

3. $AB\bar{D} + B\bar{C}\bar{D} + ACD$

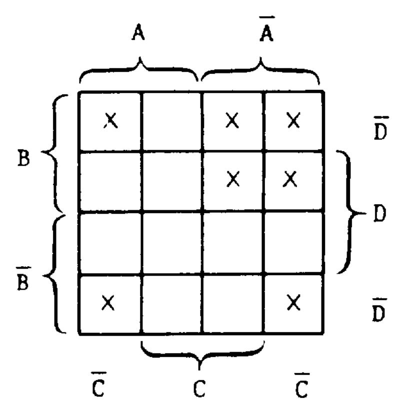
4. $\bar{A}\bar{B}CD + AB\bar{C}\bar{D} + ABCD$

ANSWERS:

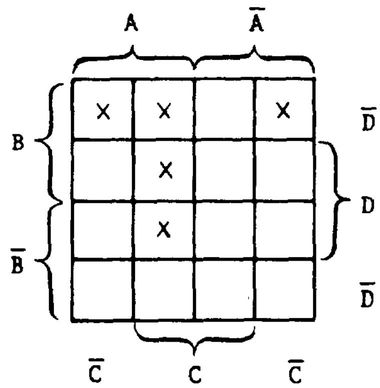
1.



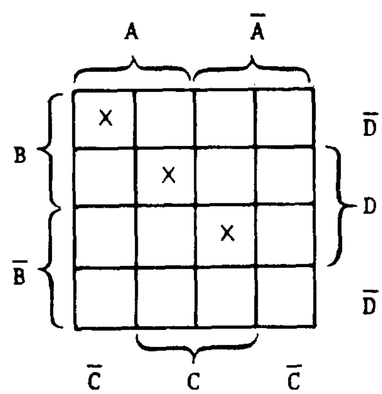
2.



3.



4.



When extracting the simplest expression from a 16 square Veitch diagram, we use the same principles that we used on an 8 square diagram. The difference is that we now look for the following:

1. Eight plotted squares described by a one-variable term.

2. Four plotted squares described by a two-variable term.

3. Two plotted squares described by a three-variable term.

4. One plotted square described by a four-variable term.

The following are examples of Veitch diagrams to illustrate patterns which should be recognized. Generally these patterns are formed by either adjacent squares or squares on the opposite ends of rows or columns.

Examples of squares at opposite ends of rows or columns are shown in figure 8-1.

Examples of adjacent squares are shown in figure 8-2.

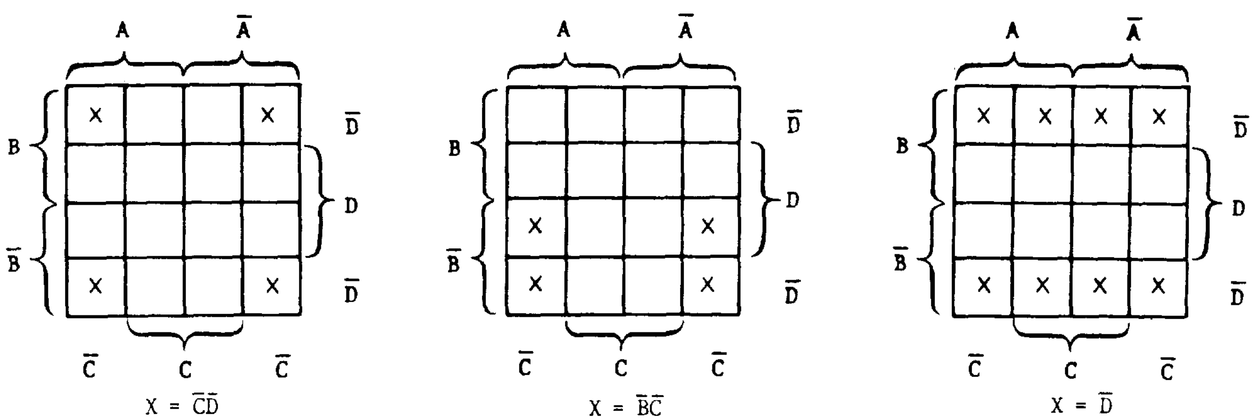


Figure 8-1.—Squares at opposite ends of rows or columns.

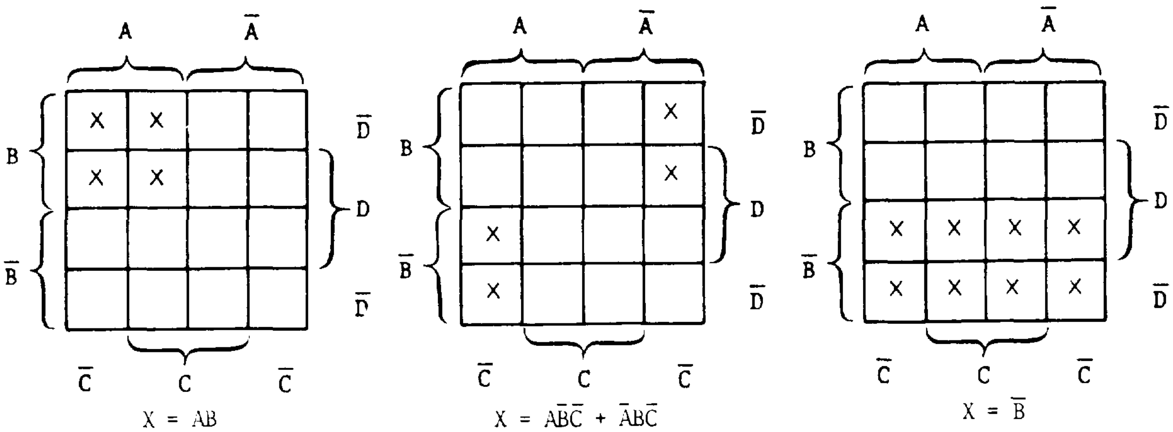
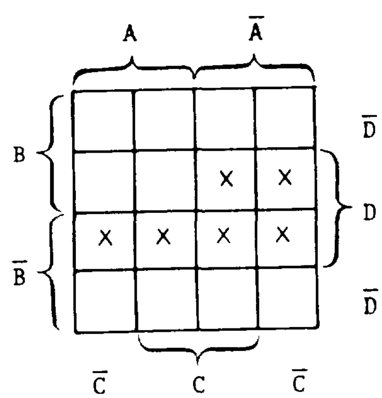


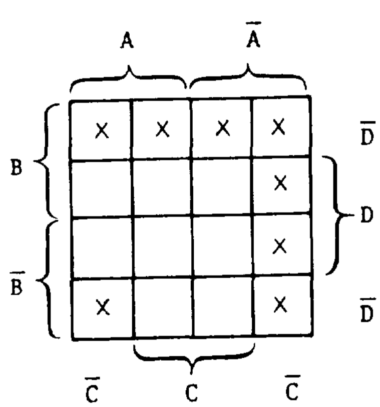
Figure 8-2.—Adjacent squares.

PROBLEMS: Describe the following plots as simply as possible.

1.



2.

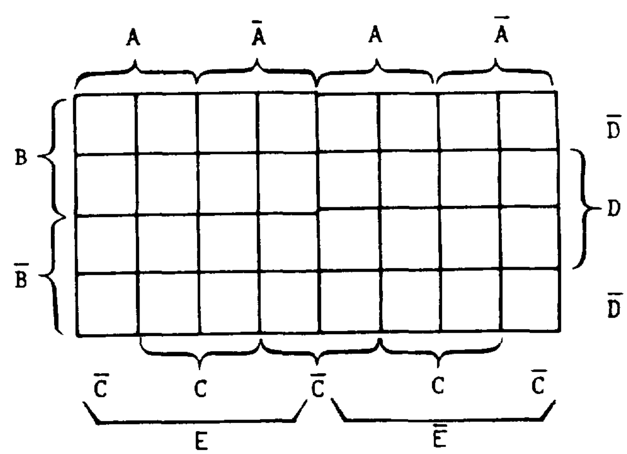


ANSWERS:

1. $\bar{B}\,D + \bar{A}\,D$

2. $\bar{D}\,\bar{C} + B\,\bar{D} + \bar{A}\bar{C}$

When we are faced with a five-variable expression, we use 2^5 squares. We label the Veitch diagram as

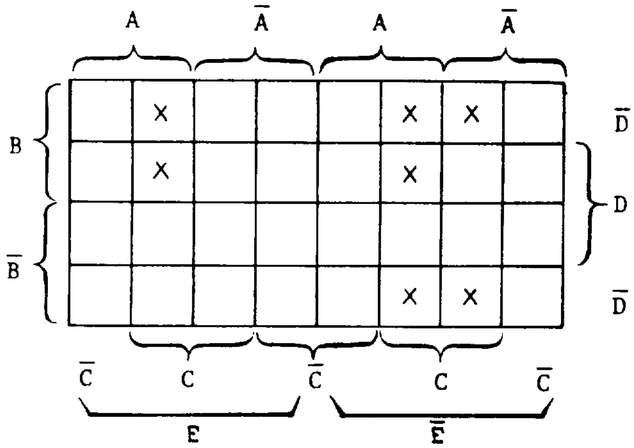


In the 32 square diagram we find a one-variable term described by 16 squares, a two-variable term described by 8 squares, a three-variable term by 4 squares, a four-variable term by 2 squares, and a five-variable term by 1 square.

To plot the expression

$$ABC + C\bar{D}\bar{E}$$

we write



In order to simplify an expression using a Veitch diagram, we follow the same procedure as before; that is, to simplify the expression

$$E(AC + B\bar{C}) + \overline{B + C + D + E}$$

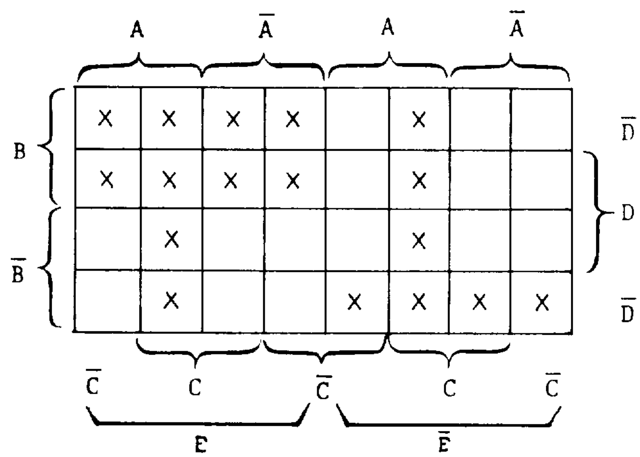
$$+ C(BE + A\bar{B} + \overline{A + B + D + E}) + \overline{\bar{A} + \bar{B} + \bar{C}}$$

we use the laws of Boolean algebra to write the minterm expression

$$EAC + EB\bar{C} + \bar{B}\bar{C}\bar{D}\bar{E} + CBE$$

$$+ CA\bar{B} + C\bar{A}\bar{B}\bar{D}\bar{E} + ABC$$

then plot the Veitch diagram as



We then extract the simplest expression, as

$$BE + AC + \bar{B}\bar{D}\bar{E}$$

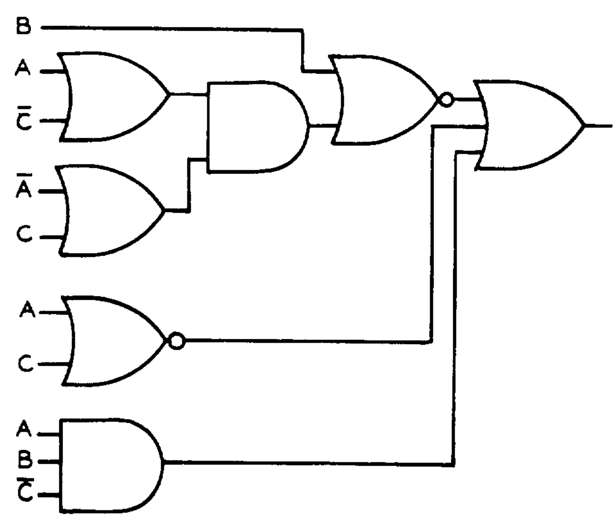
BOOLEAN EXPRESSIONS AND LOGIC DIAGRAMS

The previous sections have dealt with simplification of expressions, plotting Veitch diagrams, and extracting the simplest expressions from Veitch diagrams. In order to see the total value of these functions, we will determine their results by the step-by-step application of simplification.

If we have the expression

$$\overline{B + (A + \bar{C})(\bar{A} + C)} + \overline{A + C} + AB\bar{C}$$

we may draw the logic diagram



We may question whether this diagram is constructed using the fewest gates possible. To determine this, we employ the laws of Boolean algebra to change the given expression to minterm form. We write

$$\overline{B + (A + \bar{C})(\bar{A} + C)} + \overline{A + C} + AB\bar{C}$$

$$= \bar{B}\bar{A}C + \bar{B}A\bar{C} + \bar{A}\bar{C} + AB\bar{C}$$

then plot these terms on a Veitch diagram as

	A	A	$\bar{A}$	$\bar{A}$
B	X			X
$\bar{B}$	X		X	X
	$\bar{C}$	C	C	$\bar{C}$

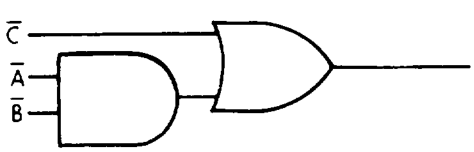
and upon extracting the simplest expression from the plotted squares, we find it to be

$$\bar{A}\bar{B} + \bar{C}$$

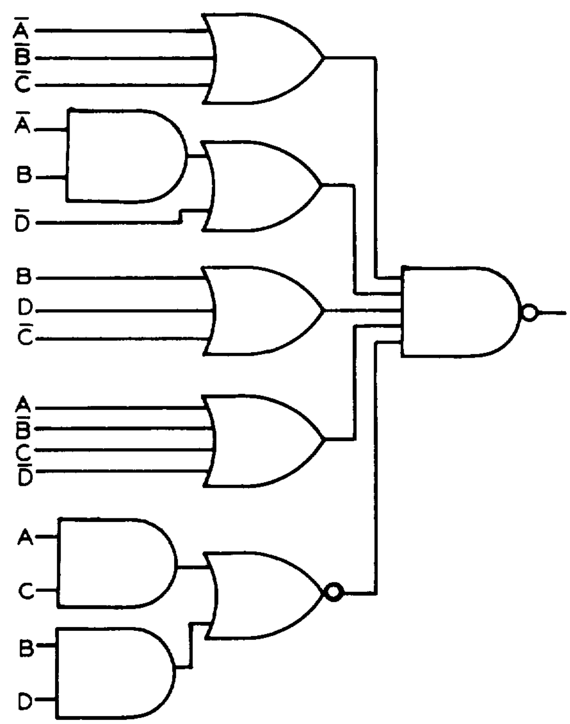
Our next step is to draw the logic diagram for

$$\bar{A}\bar{B} + \bar{C}$$

which is



Another example of this technique of simplification is given using the logic diagram



which is identified by the expression

$$\overline{(\bar{A} + \bar{B} + \bar{C})(\bar{A}B + \bar{D})(B + D + \bar{C})}$$

$$\overline{(A + \bar{B} + C + \bar{D})\overline{(AC + BD)}}$$

When written in minterm form, this is

$$ABC + AD + \bar{B}D + \bar{B}\bar{D}C + \bar{A}B\bar{C}D + AC + BD$$

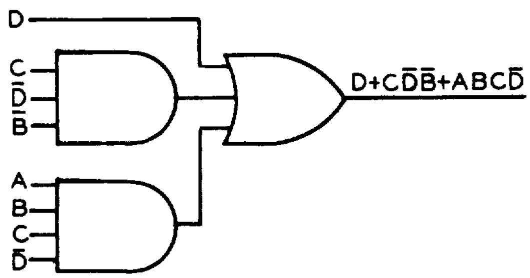
and when plotted, it appears as

		A	A	$\bar{A}$	$\bar{A}$	
B			X			$\bar{D}$
B		X	X	X	X	D
$\bar{B}$		X	X	X	X	D
$\bar{B}$			X	X		$\bar{D}$
		$\bar{C}$	C	C	$\bar{C}$	

Simplification of these squares results in

$$D + C\bar{D}\bar{B} + ABC\bar{D}$$

which has the logic diagram of



This may be further simplified as